UNIVERSITY OF ILLINOIS
AT URBANA-CHAMPAIGN

Pulses in transmission lines

Physics 401, Spring 2013
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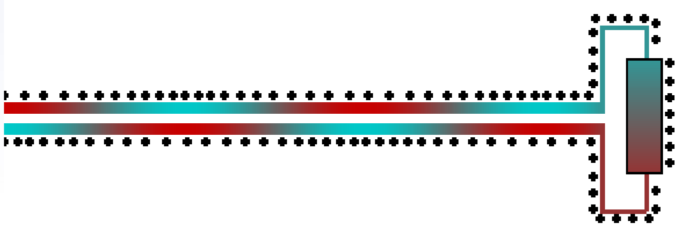
illinois.edu

Transmission lines.
Agenda.

- Definition
- Distributed parameters network
- Pulses in transmission line
- Wave equation and wave propagation
- Reflections. Resistive load
- Thévenin's theorem
- Reflection. Non resistive load
- Appendix. Error propagation

Transmission lines. Distributed parameters network.

- **Transmission line is a specialized cable designed to carry alternating current of radio frequency, that is, currents with a frequency high enough that its wave nature must be taken into account.**

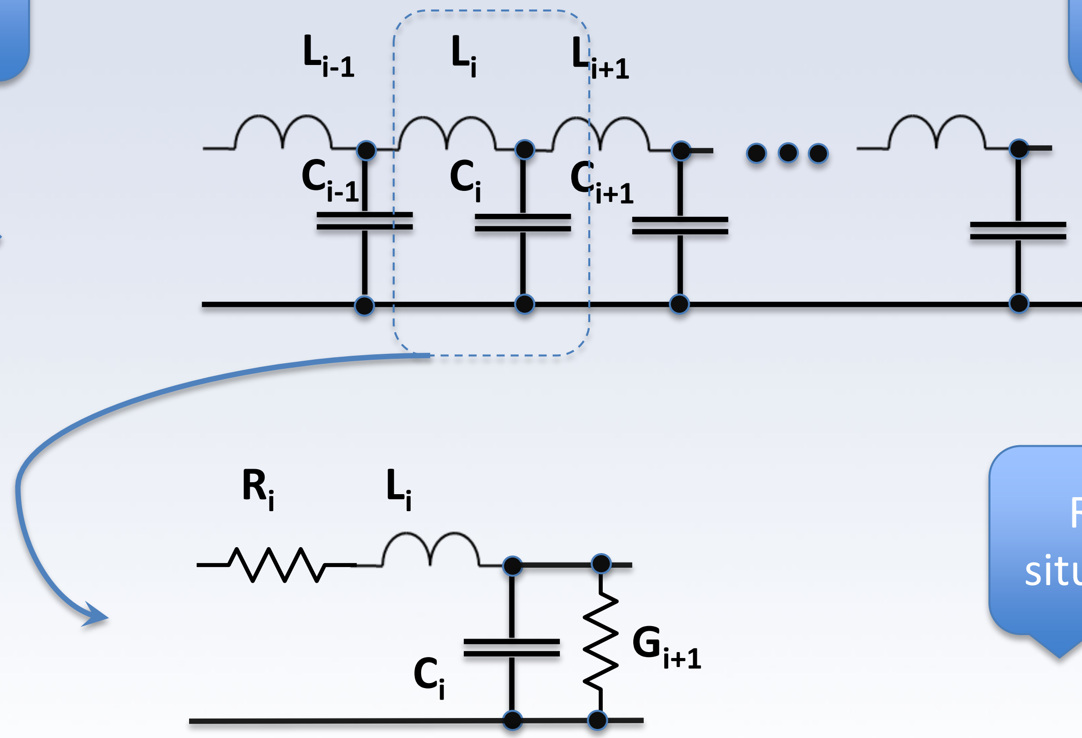


Courtesy Wikipedia

Transmission lines. Distributed parameters network

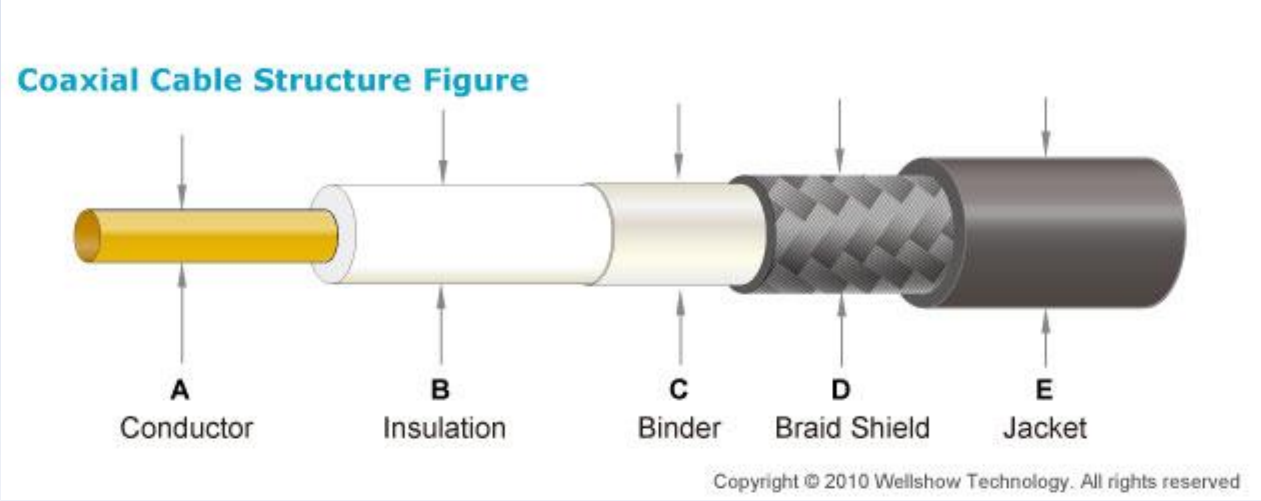
Simplified equalent circuit

Ideal case



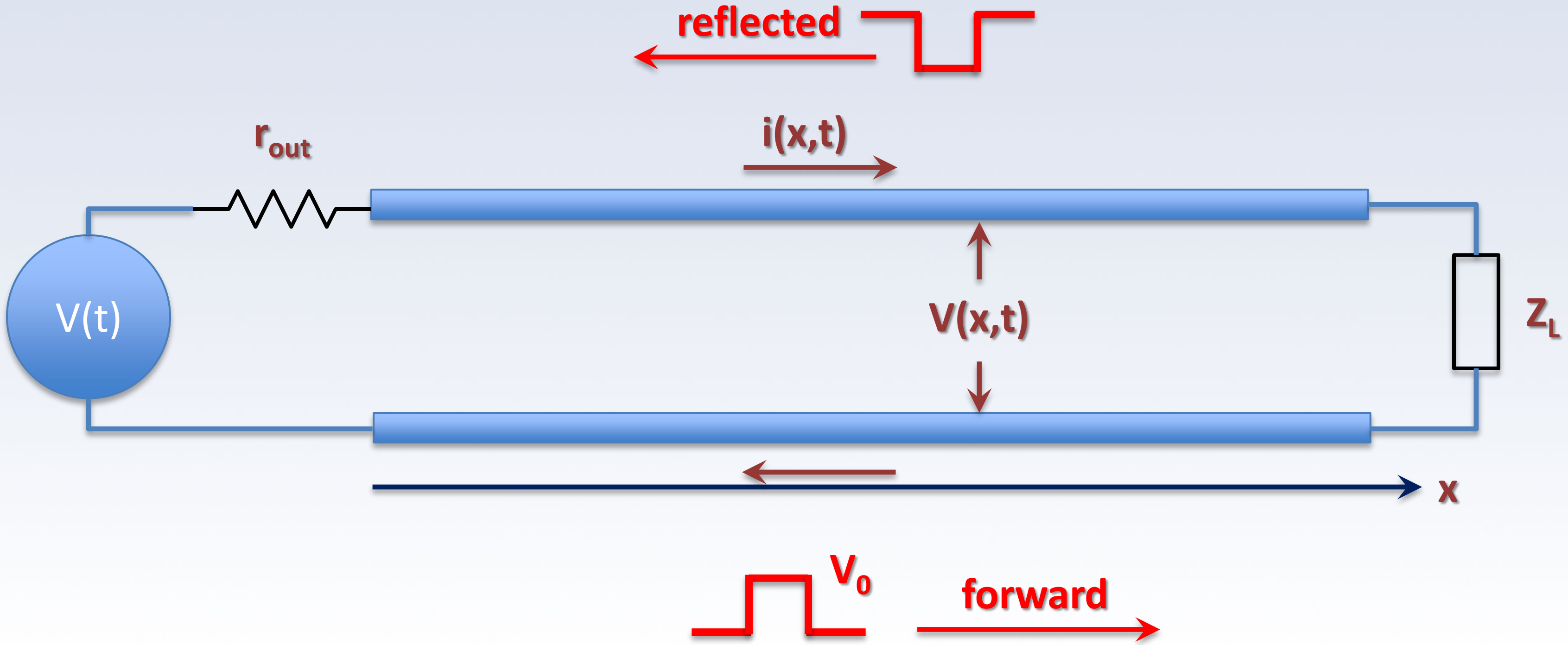
Real situation

Coaxial cable

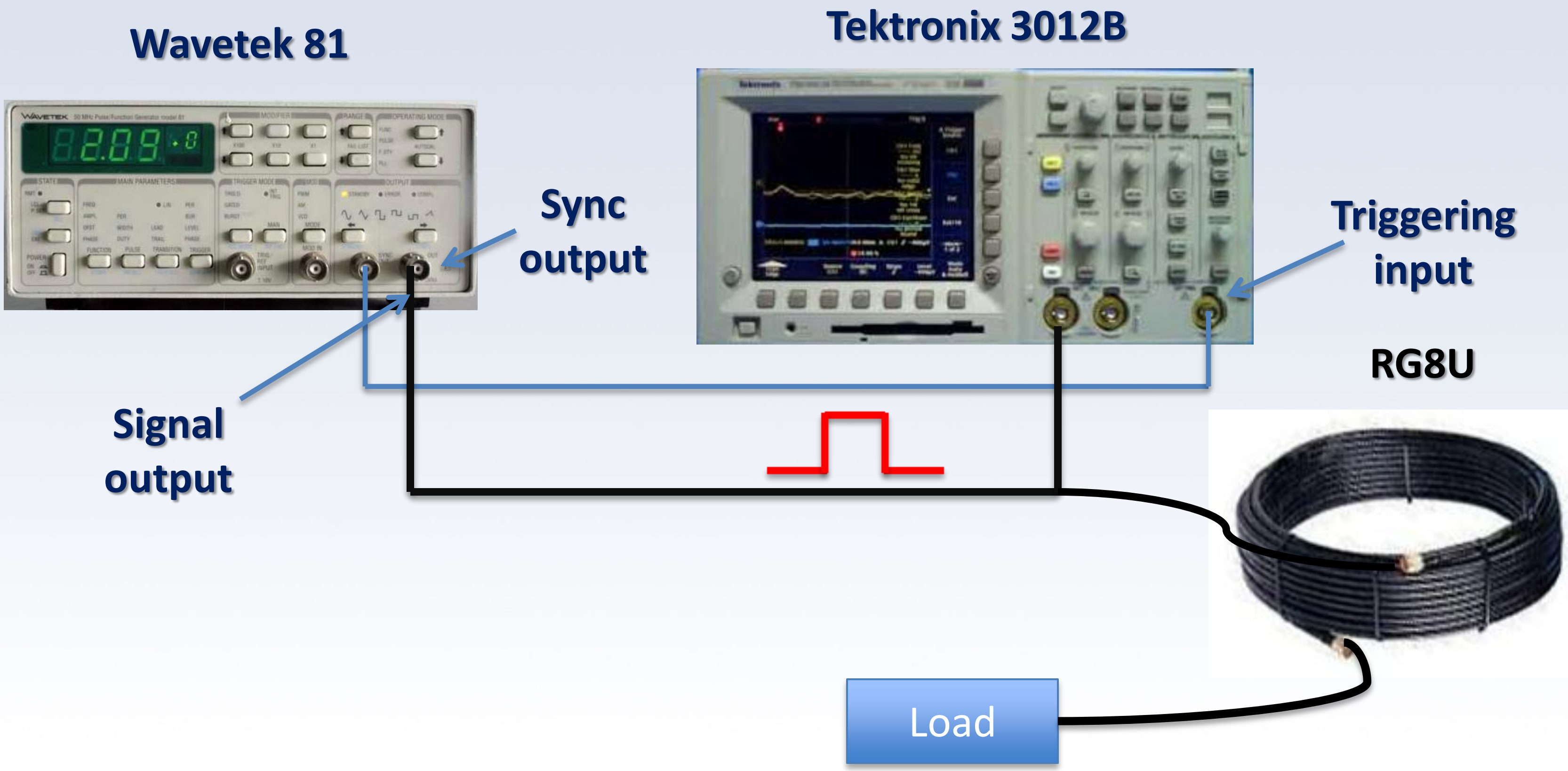


Specification:
Impedance: 53 Ω
Capacitance: 83 pF/m
Conductor: Bare Copper Wire (1/1.02mm)

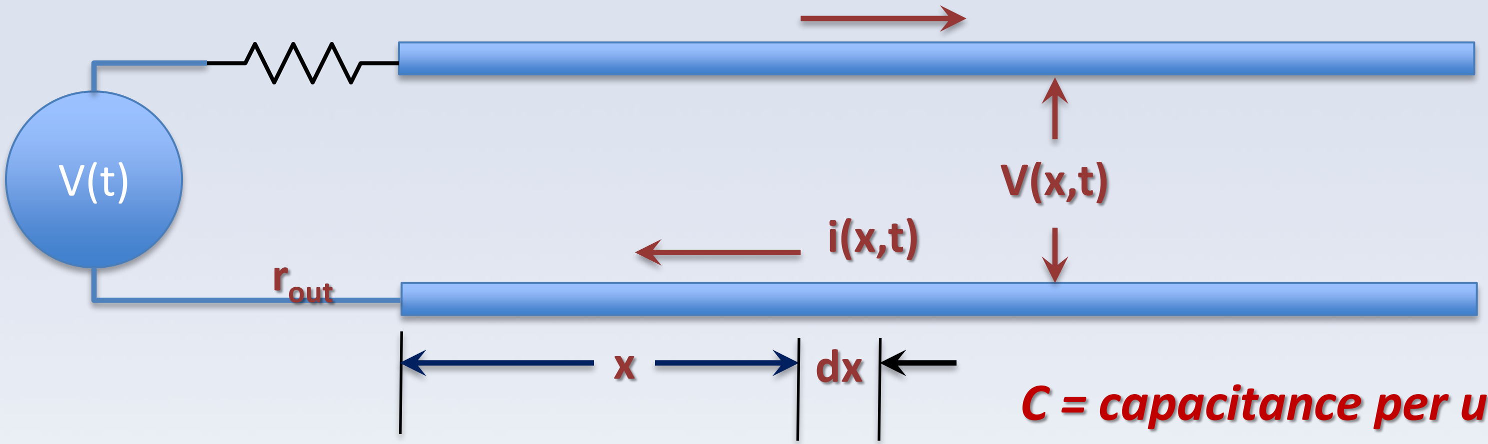
Pulses in transmission line



Setup


Wavetek 81
Tektronix 3012B
Sync output
Triggering input
RG8U
Signal output
Load

The Wave Equation



C = capacitance per unit length

L = inductance per unit length

$$CdxV = -dq;$$

$$C\frac{\partial V}{\partial t} = -\frac{\partial q}{\partial t} = i;$$

$$\frac{\partial i}{\partial x} = -C\frac{\partial V}{\partial t}$$

$$dV = -(Ldx)\frac{di}{dt};$$

$$\frac{\partial V}{\partial x} = -L\frac{\partial i}{dt}$$

The Wave Equation

$$\frac{\partial i}{\partial x} = -C\frac{\partial V}{\partial t}$$

$$\frac{\partial V}{\partial x} = -L\frac{\partial i}{dt}$$

$\frac{\partial}{\partial t}$ ⇓ $\qquad$ $\frac{\partial}{\partial x}$ ⇓

$$\frac{\partial^2 i}{\partial t \partial x} = -C\frac{\partial^2 V}{\partial t^2} \qquad (1)$$

$$\frac{\partial^2 V}{\partial x^2} = -L\frac{\partial^2 i}{\partial x \partial t} \qquad (2)$$

Combining (1) and (2)

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2} \qquad \frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2}$$

The Wave Equation. Voltage and current waves.

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2} \qquad \frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2}$$

Looking for solution

$$V(x,t) = V_0 \sin\omega\left(t - \frac{x}{v}\right)$$

$$i(x,t) = i_0 \sin\omega\left(t - \frac{x}{v}\right)$$

Now substituting *V(x,t)* and *i(x,t)* in

$$\frac{\partial V}{\partial x} = -L\frac{\partial i}{dt} \qquad \frac{\partial i}{\partial x} = -C\frac{\partial V}{\partial t}$$

We can find $V_0 = i_0\sqrt{\frac{L}{C}}$ or

$$v = \frac{1}{\sqrt{LC}}$$

Speed of wave propagation

$$V(x,t) = \sqrt{\frac{L}{C}}\,i(x,t) = Z_k i(x,t)$$

Z_k - characteristic Impedance

Equivalent to Ohm's law equation

Characteristic impedance

$$Z_k = \sqrt{\frac{L}{C}}$$

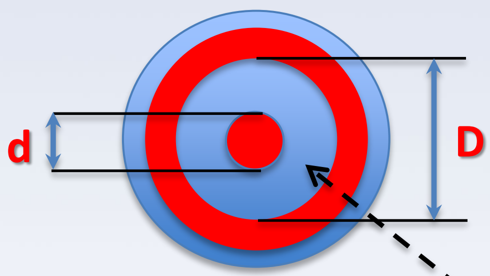
C = capacitance per unit length

L = inductance per unit length

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{(F/m)}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{(H/m)}$$

Cross-section of the coaxial cable



$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{D}{d}\right)} \quad \text{(F/m)}$$

$$L = \frac{\mu_0\mu_r}{2\pi}\ln\left(\frac{D}{d}\right) \quad \text{(H/m)}$$

ε_r – dielectric permittivity

μ_r-magnetic permeability ≈1

Finally for coaxial cable: $$Z_k = \frac{138}{\sqrt{\varepsilon_r}}\log\left(\frac{D}{d}\right)(Ohms)$$

Speed of wave propagation, delay.

$$v = \frac{1}{\sqrt{LC}}$$

Speed of wave propagation

$$v = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \approx \frac{c}{\sqrt{\varepsilon_r}}$$

$\mu_r \cong 1$

For polyethylene ε_r~2.25(up to 1GHz)

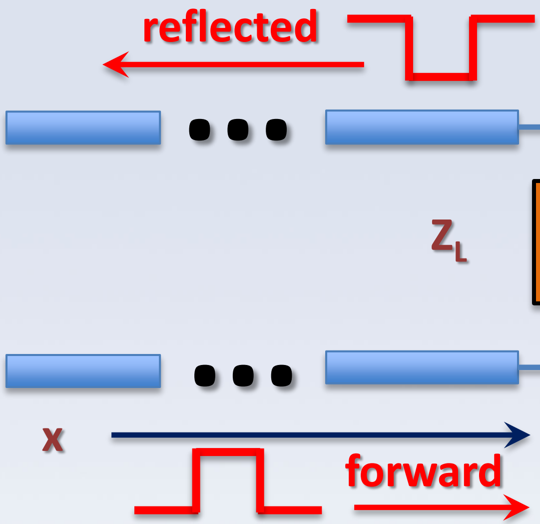
Delay time $\tau = \frac{1}{v}$(s/m) ≈ $3.336 \bullet 10^{-9} \sqrt{\varepsilon_r}\,(s/m) = 3.336\sqrt{\varepsilon_r}\,(ns/m)$

RG-8/U, RG58U:

Inner Insulation Materials: Polyethylene
Nominal Impedance: 52 ohm
Delay time ~5ns/m

Reflection in transmission line

solution for the traveling in opposite direction



$$\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2}$$

$$V(x,t) = V_0 \sin\omega\left(t - \frac{x}{v}\right)$$

$$i(x,t) = i_0 \sin\omega\left(t - \frac{x}{v}\right)$$

For reflected wave $V_r = -Z_k i_r$

At any point of the transmission line:

$$V = V_r + V_i$$

$$i = i_r + i_i = \frac{V_i}{Z_k} - \frac{V_r}{Z_k}$$

1. Resistive load $Z_L = R_L$

$$\frac{V}{i} = R_L$$

$$\frac{V_i + V_r}{V_i - V_r} = \frac{R_L}{Z_k} \quad \text{or} \quad V_r = \frac{R_L - Z_k}{R_L + Z_k} V_i$$

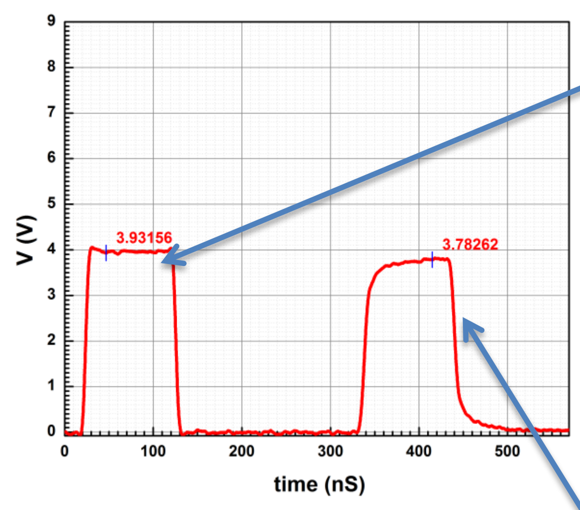
Reflection in transmission line

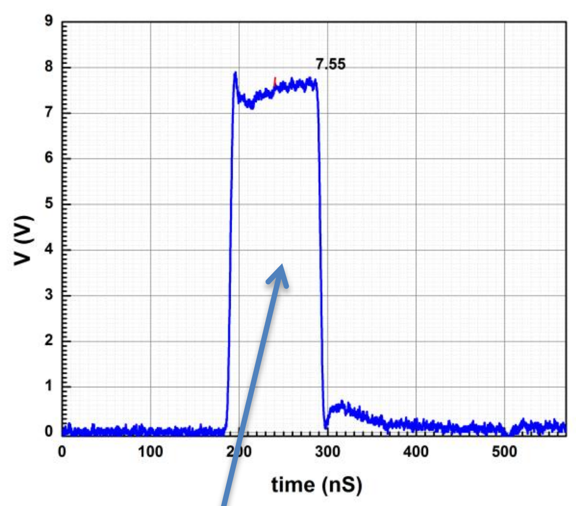
Resistive load $Z_L = R_L$

$$\frac{V_i + V_r}{V_i - V_r} = \frac{R_L}{Z_k} \quad \text{or} \quad V_r = \frac{R_L - Z_k}{R_L + Z_k} V_i$$

Open line $R_L = \infty \rightarrow V_r = V_i$

Incident pulse





Reflected pulse

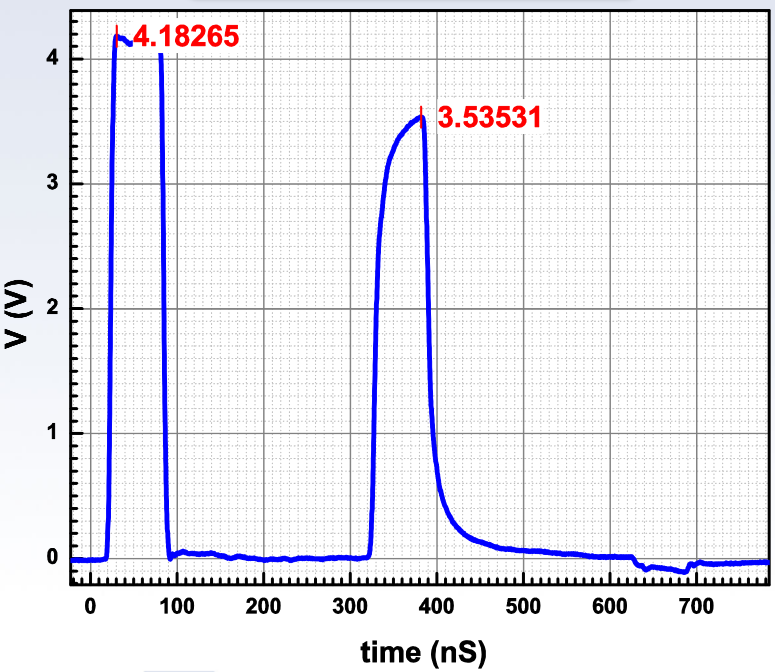
End of the line

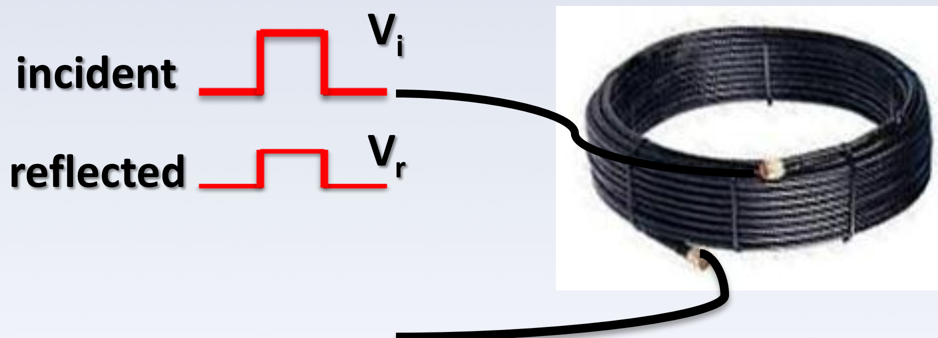
Reflection in transmission line. Loses.

Theory: $R_L = \infty \rightarrow V_r = V_i$

Attenuation (dB per 100 feet)					
MHz	30	50	100	146	150
RG-58U	2.5	4.1	5.3	6.1	6.1

Experiment RG 58U

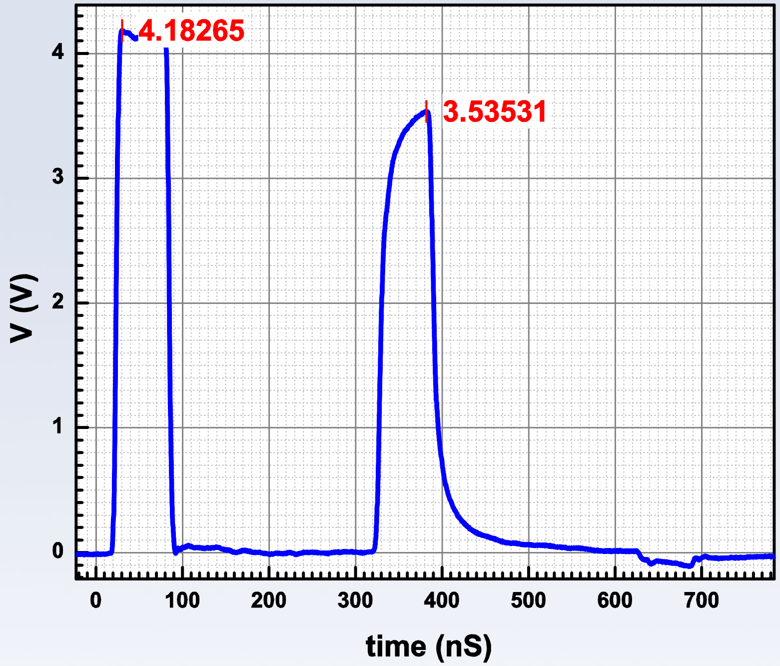




$$ATTN(db) = 20\log\left(\frac{V_i}{V_r}\right)$$

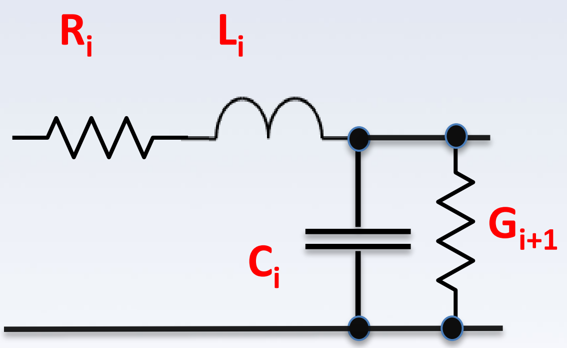
Important parameter for cable is attenuation per length

Reflection in transmission line. Loses.

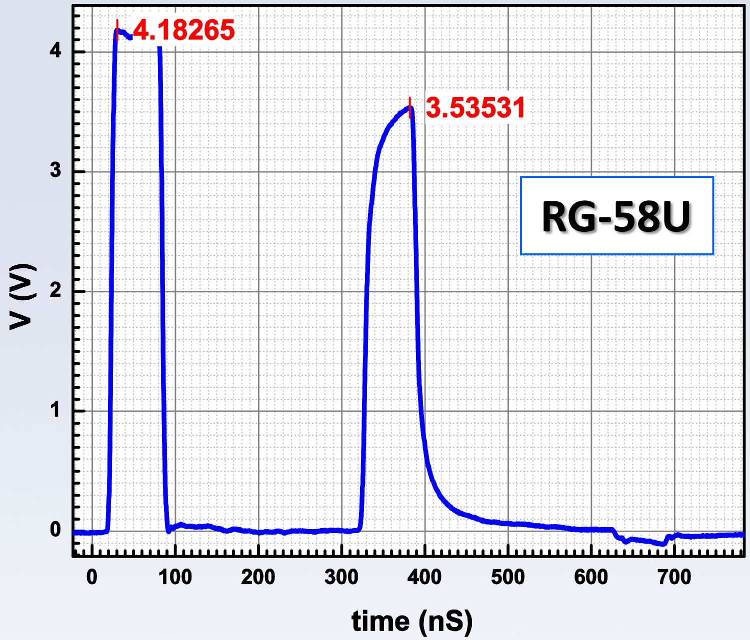


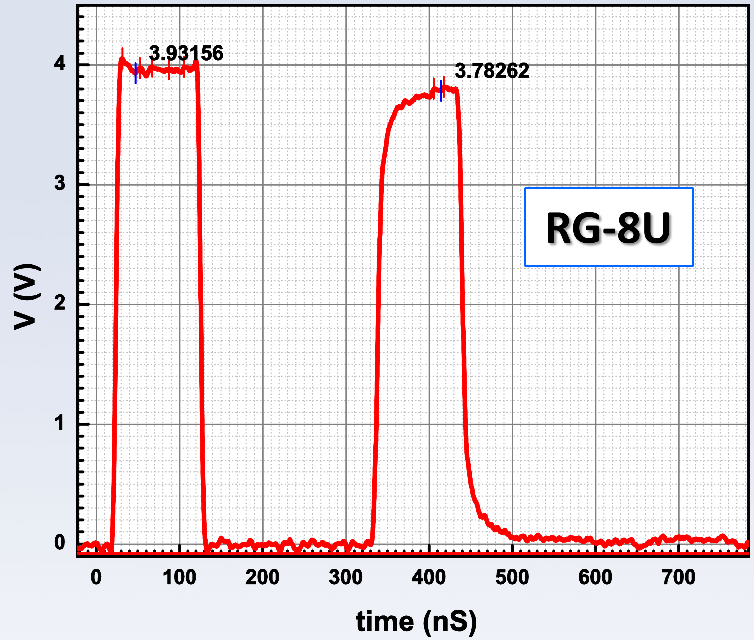
In our case: $Attn(200\,ft) = 20\log\left(\frac{4.18}{3.54}\right) \approx 1.46dB$

Where it is coming from?



Different cables loses.

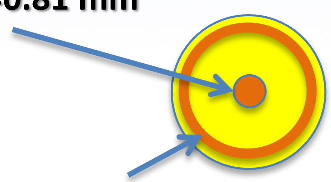


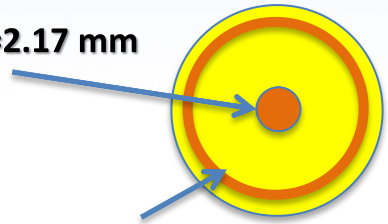


$$Attn(200\ ft) = 20\log\left(\frac{4.18}{3.54}\right) \approx 1.46dB$$

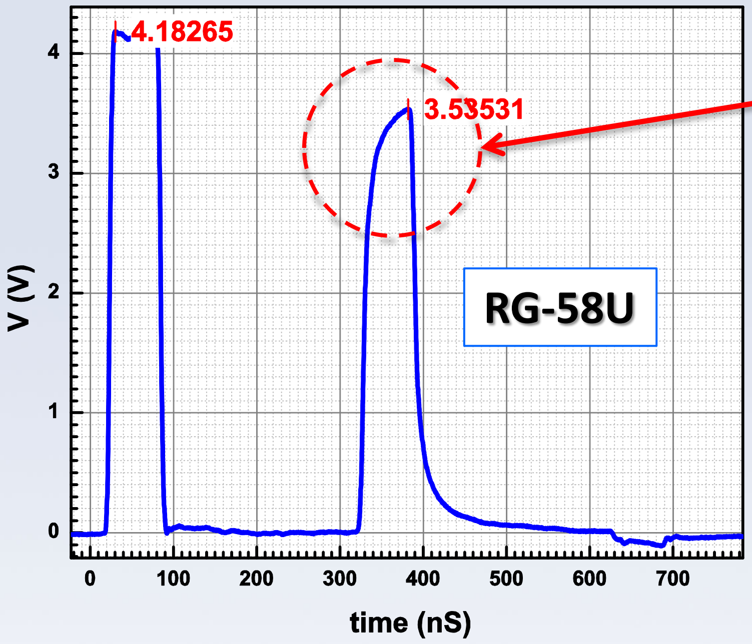
>

$$Attn(200\ ft) = 20\log\left(\frac{3.932}{3.78}\right) \approx 0.335dB$$

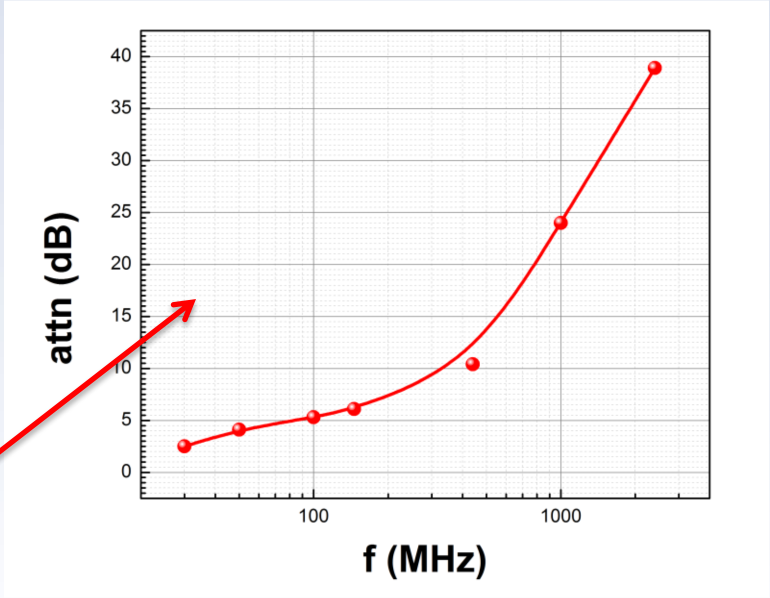




Loses. Frequency dispersion.

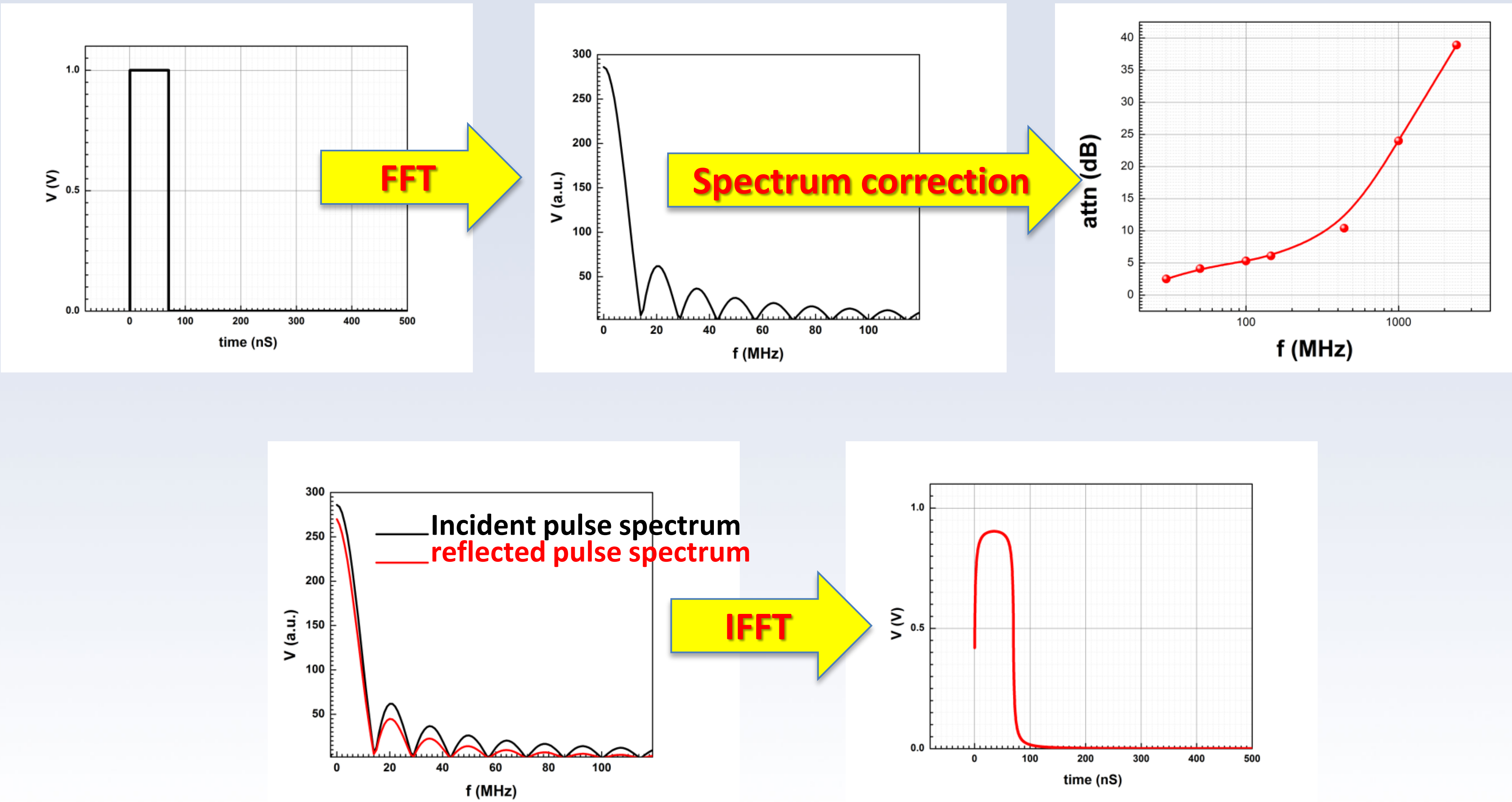


Reflected pulse does not follow the shape of the incident pulse



Frequency dependence of the attenuation RG-58U cable

Loses. Frequency dispersion.

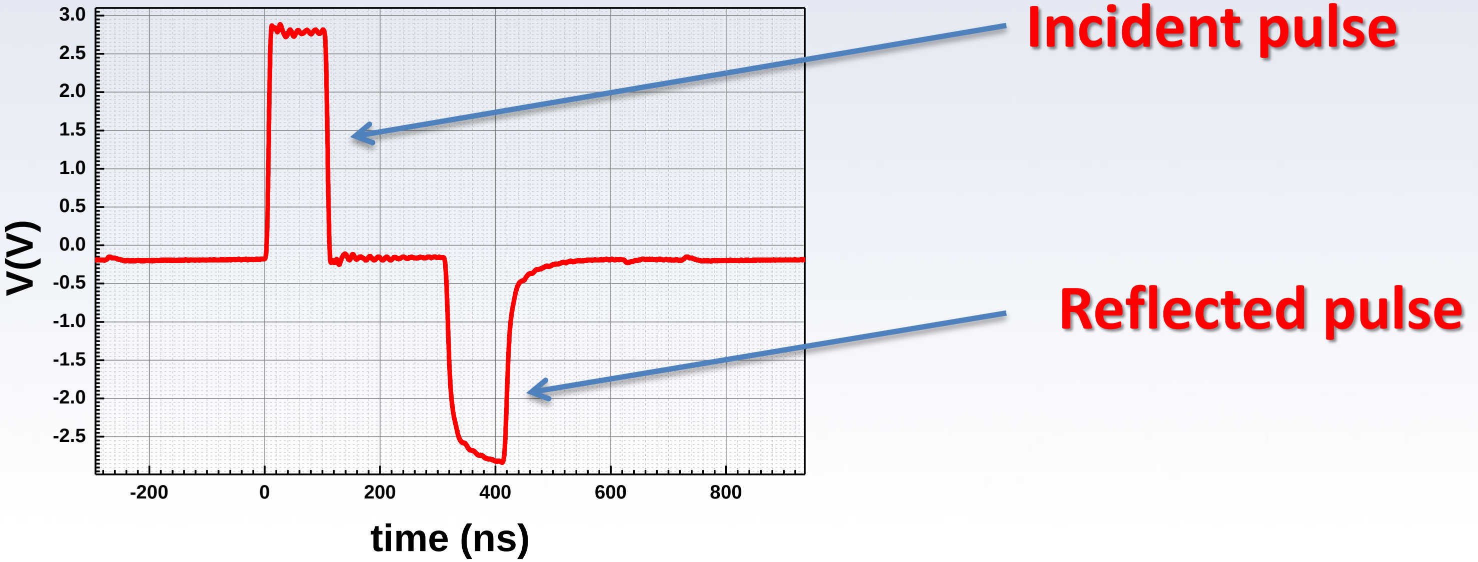


Reflection in transmission line

Resistive load $Z_L=R_L$

$$\frac{V_i + V_r}{V_i - V_r} = \frac{R_L}{Z_k} \quad \text{or} \quad V_r = \frac{R_L - Z_k}{R_L + Z_k} V_i$$

Shorted line $R_L=0$ → $V_r = - V_i$



Reflection in transmission line.

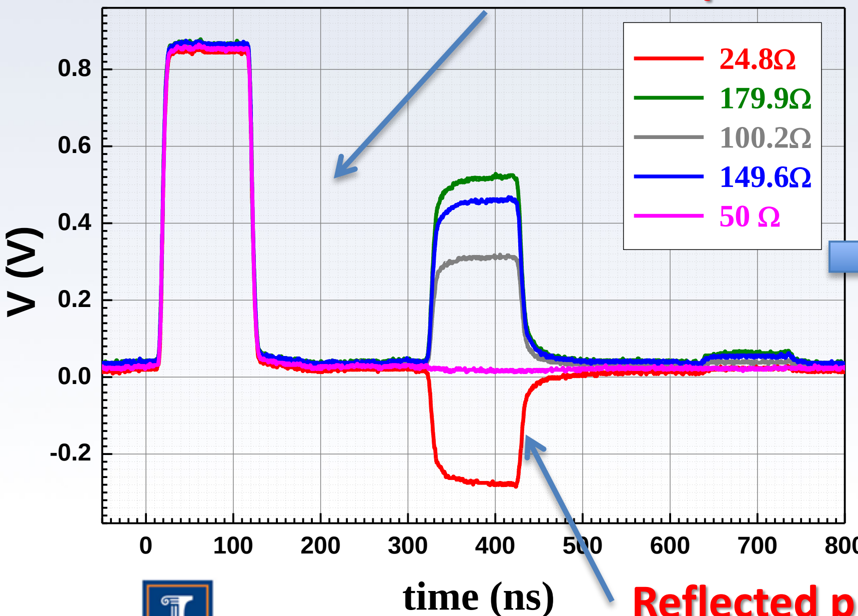
Resistive load $Z_L=R_L$

$$\frac{V_i+V_r}{V_i-V_r}=\frac{R_L}{Z_k} \quad \text{or} \quad V_r=\frac{R_L-Z_k}{R_L+Z_k}V_i$$

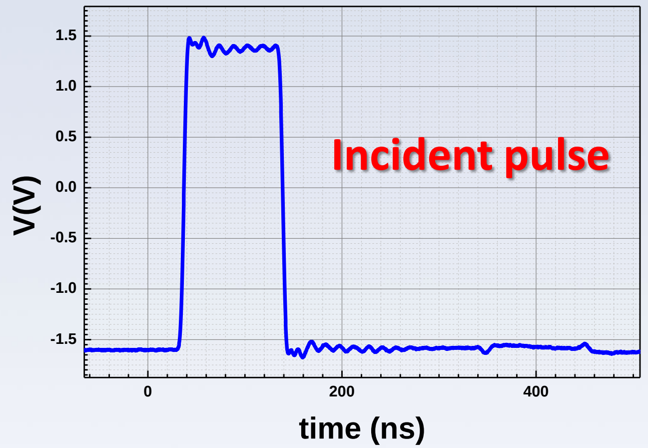
Matching the load impedance

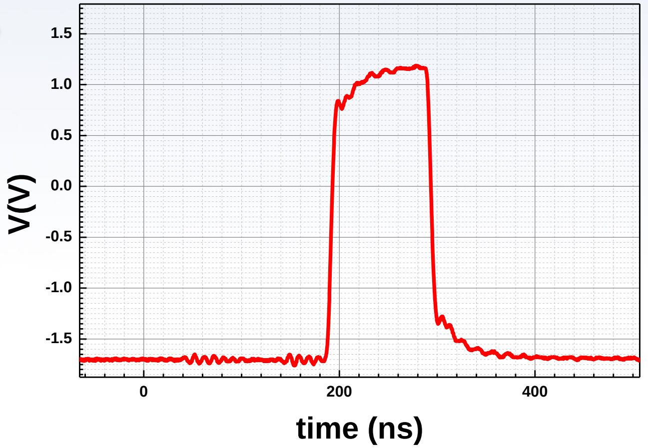
$R_L \rightarrow Z_k;\ V_r \rightarrow 0$

Incident pulse



Reflected pulse





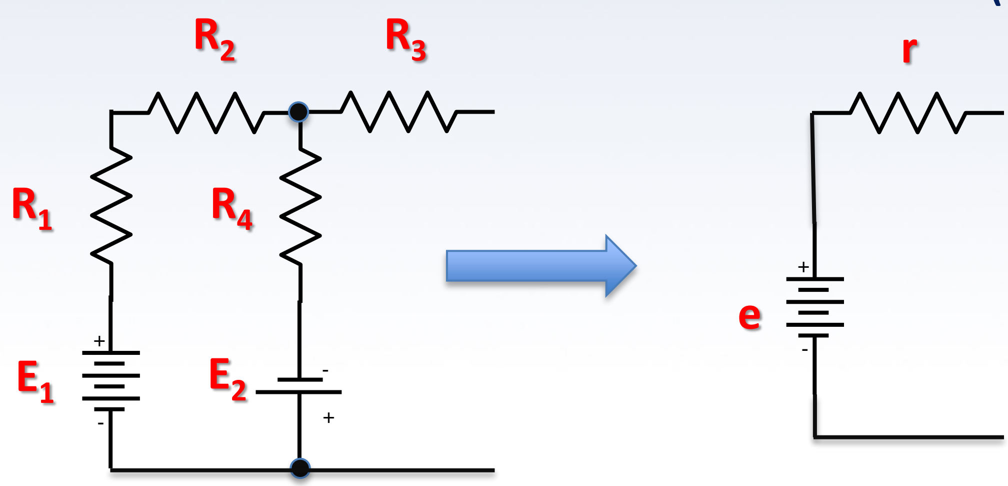
End of the line

Thévenin's theorem

Hermann Ludwig Ferdinand von Helmholtz (1821-1894)

Any combination of batteries and resistances with two terminals can be replaced by a single voltage source e and a single series resistor r

Léon Charles Thévenin (1857–1926)

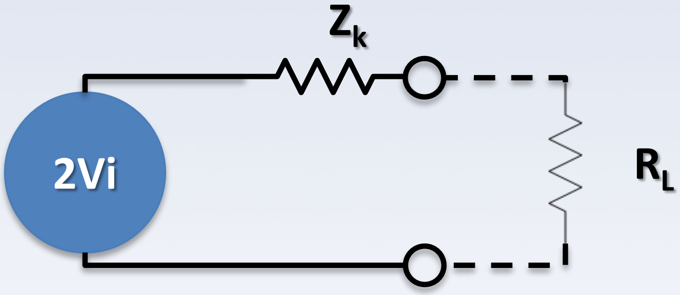


Thévenin's theorem. Transmission line.

$$V = V_r + V_i = i \bullet R_L$$

$$i = i_r + i_i = \frac{V_i}{Z_k} - \frac{V_r}{Z_k}$$

$$i = \frac{2V_i}{R_L + Z_k}$$

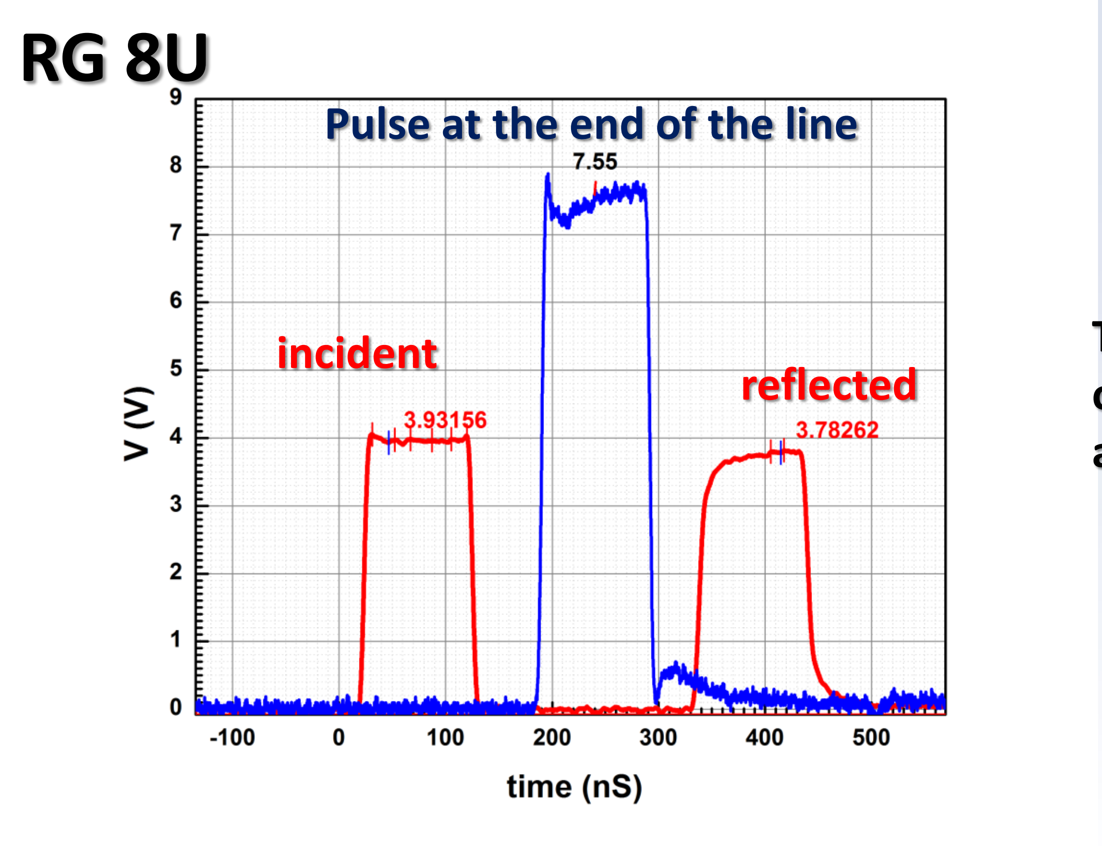


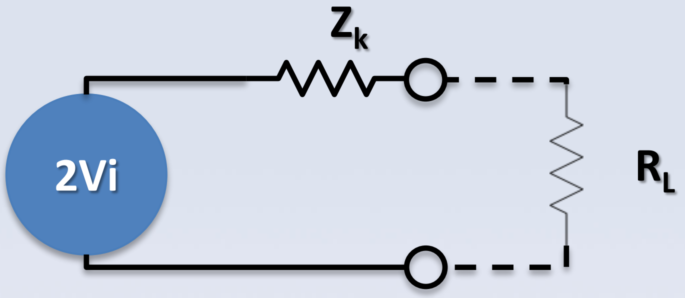
From this equivalent equation we can find the maximum possible power delivered to R_L

$$P = i^2 R_L = \frac{(2V_i)^2}{(R_L + Z)^2} R_L$$

$P = P_{max}$ if $R_L = Z_k$ (no reflection)

Thévenin's theorem. Experiment.



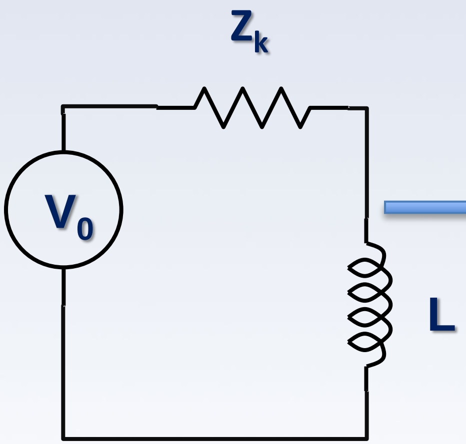


This experiment better to perform on RG 8U cable because of lower attenuation

RL=∞, amplitude of the pulse at the end of line is expected to be 2Vi, where Vi is the amplitude of the incident pulse

Reflection. Inductive load.

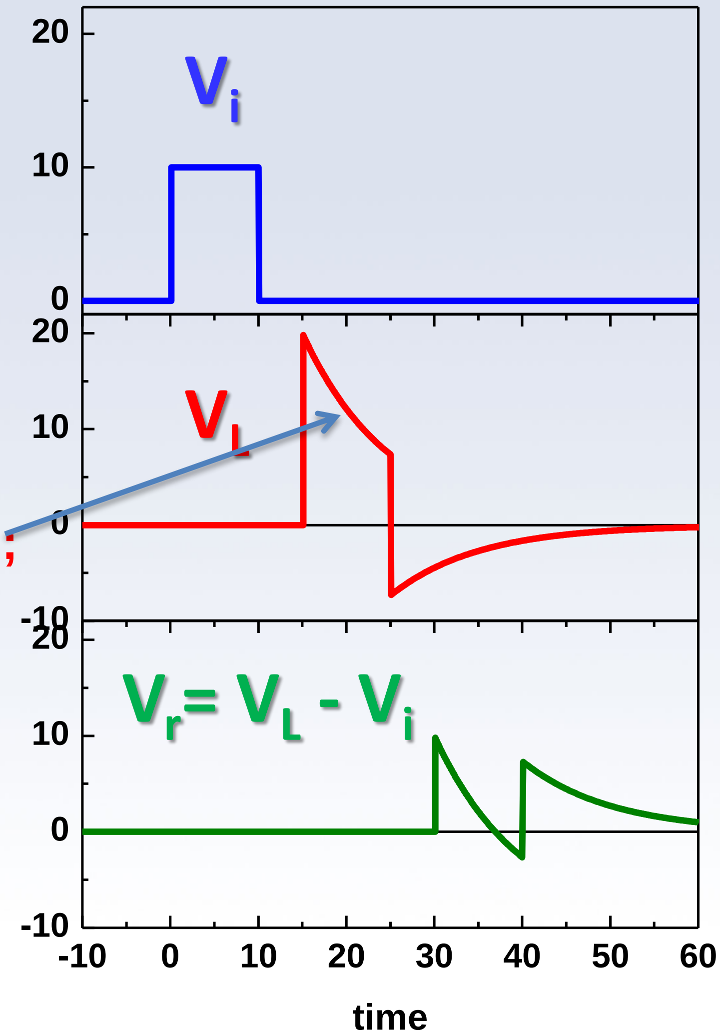
$$i = \frac{2V_i}{Z_L + Z_k}$$



$$2V_i = iZ_k - L\frac{di}{dt};$$

$$i = i_0\left(1 - \exp\left(\frac{-t}{\tau}\right)\right);$$

$$\tau = \frac{L}{Z_k}$$

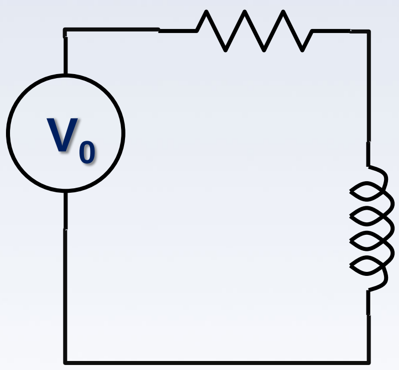


Reflection. Inductive load. Experiment

$$i = \frac{2V_i}{Z_L + Z_k}$$

$$2V_i = iZ_k - L\frac{di}{dt};$$

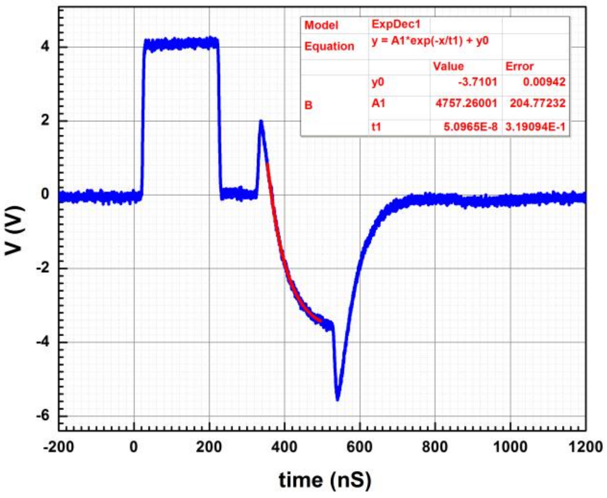
$$i = i_0\left(1 - \exp\left(\frac{-t}{\tau}\right)\right);$$

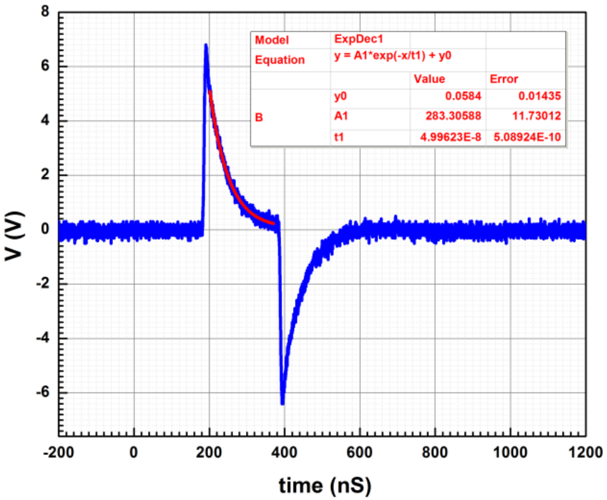


$$\tau = \frac{L}{Z_k}$$

$\tau \cong 50\text{ns},$

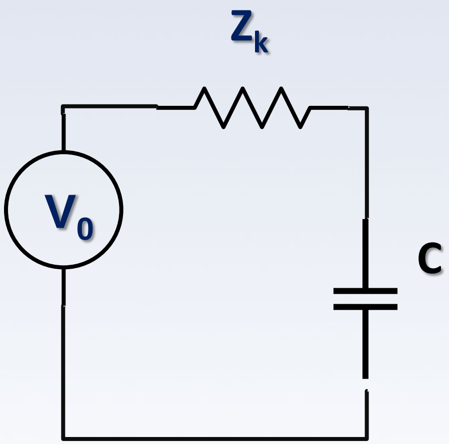
$L = \tau Zk \sim 2.5\mu H$





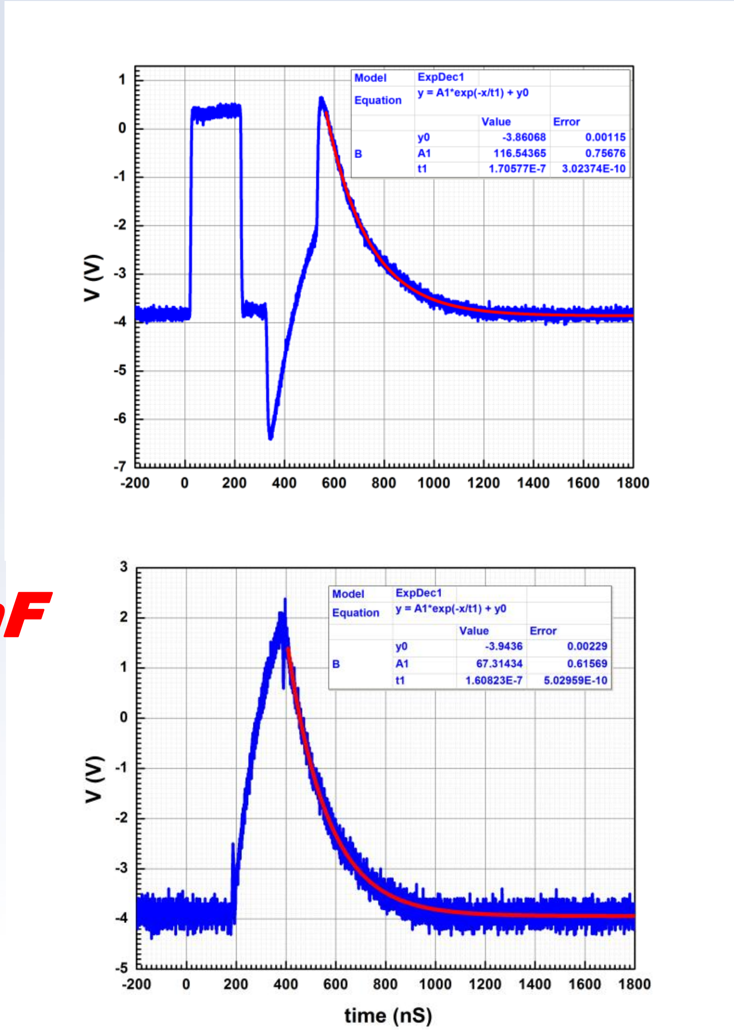
Reflection. Capacitive load. Experiment

$$i = \frac{2V_i}{Z_L + Z_k}$$

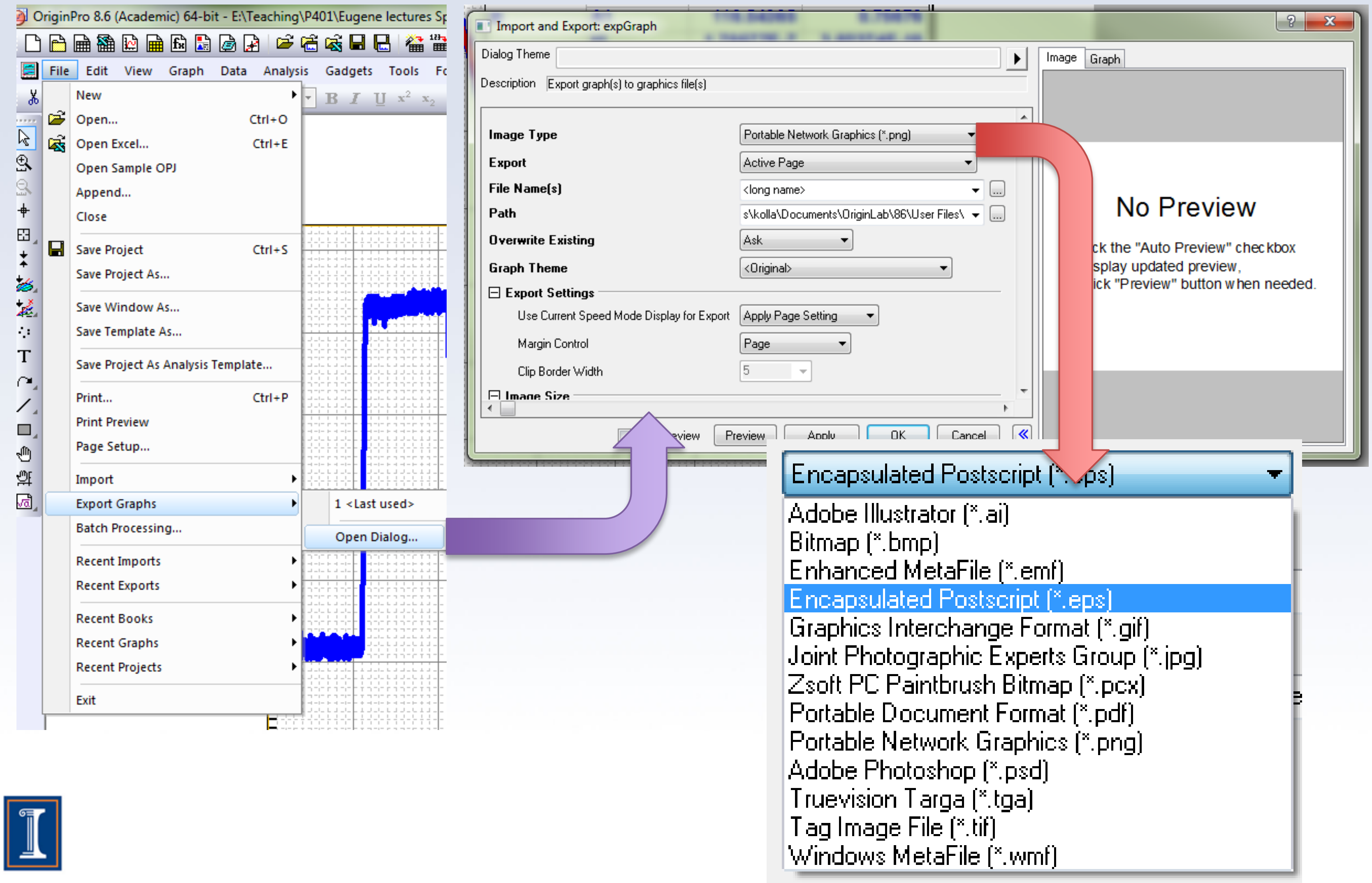


$$\tau = Z_k C$$

$$C = \frac{\tau}{Z_k} \approx 3.2nF$$



Appendix #1. Export graphs from Origin



Appendix #2. Reminders

1. The reports should be uploaded to the ***proper*** folder and ***only*** to the proper folder

For example folder ***RLC Lab Report_L1*** should used by students from L1 section only

I would recommend the file name style as:

L1_lab2_student1

Lab section Lab number Your name

2. Origin template for this week Lab:

\\Phyaplportal\PHYCS401\Common\Origin templates\Transmission line\Time trace.otp

Appendix #3. RLC resonance circuit. Fitting.

R_{limit} –s a serious to LCR circuit resistor

Fitting parameters:

(1) $\frac{R}{R_{limit}}$, (2) $\omega_0 = \frac{1}{\sqrt{LC}}$,

(3) $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

Initial values can be estimated from circuit parameters.

$$\tilde{H} = \left(\frac{1}{R_{limit}}\right)\frac{R + j\omega L}{\left(1 - \frac{\omega^2}{\omega_0^2}\right) + j\omega RC} =$$

$$\left(\frac{R}{R_{limit}}\right)\frac{1 + j\frac{\omega}{\omega_0}Q}{\left(1 - \frac{\omega^2}{\omega_0^2}\right) + j\frac{\omega}{\omega_0}\frac{1}{Q}}$$

$$= \left(\frac{R}{R_{limit}}\right)\frac{1 - j\frac{\omega}{\omega_0}\left(\frac{1}{Q} - Q\left(1 - \frac{\omega^2}{\omega_0^2}\right)\right)}{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \left(\frac{\omega}{\omega_0}\right)^2\frac{1}{Q^2}}$$

For fitting results and actual fitting function equation go to:
Phyaplportal\PHYCS401\Common\SimpleExamples\
Lab 3 Frequency Domain Analysis_example.opj

Now to simplify the equation we can introduce the reduced frequency $\gamma = \frac{\omega}{\omega_0}$ and transfer function components can be presented as:

$$\tilde{H}(\omega) = \left(\frac{R}{R_0} \bullet Q\right) \frac{Q - j\gamma\left(1 - Q^2\left(1-\gamma^2\right)\right)}{Q^2\left(1-\gamma^2\right)^2 + (\gamma)^2};$$

$$H_{RE} = \left(\frac{R}{R_0} \bullet Q\right) \frac{Q}{Q^2\left(1-\gamma^2\right)^2 + (\gamma)^2};$$

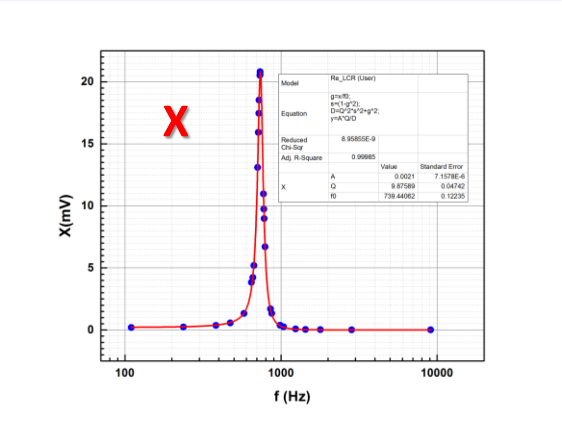
$$H_{IM} = \left(\frac{R}{R_0} \bullet Q\right) \frac{-j\gamma\left(1 - Q^2\left(1-\gamma^2\right)\right)}{Q^2\left(1-\gamma^2\right)^2 + (\gamma)^2}$$

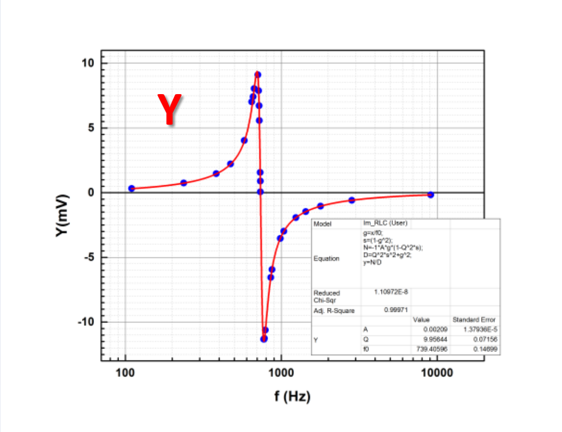
Two other components the modulus and the phase shift:

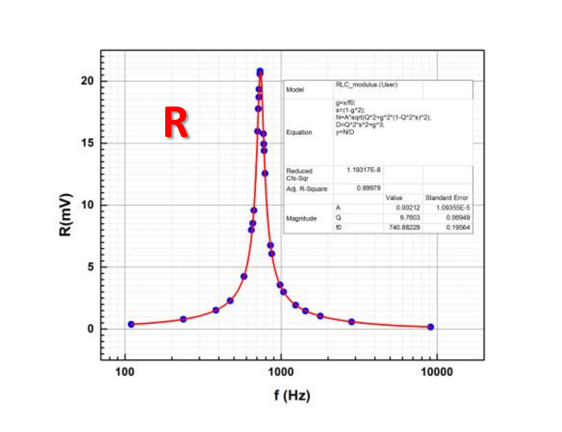
$$\left|\tilde{H}\right| = \left(\frac{R}{R_0}\bullet Q\right)\frac{\sqrt{Q^2 +^2\left(1-Q^2\left(1-\gamma^2\right)\right)^2}}{Q^2\left(1-\gamma^2\right)^2+\left(\gamma\right)^2};$$

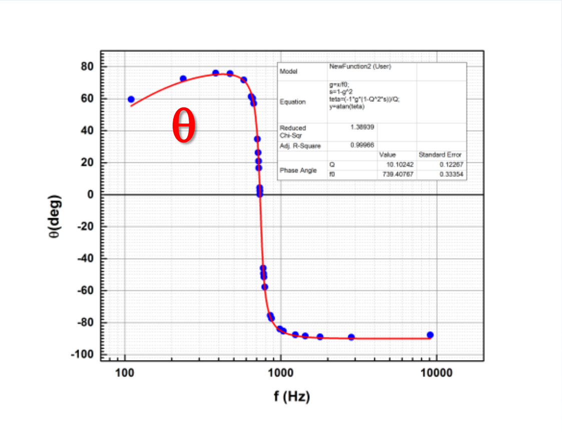
$$\theta = \arctan\left(\frac{H_{IM}}{H_{RE}}\right) = \arctan\left(\frac{-\gamma\left(1-Q^2\left(1-\gamma^2\right)\right)}{Q}\right)$$

Appendix #3. Fitting results.







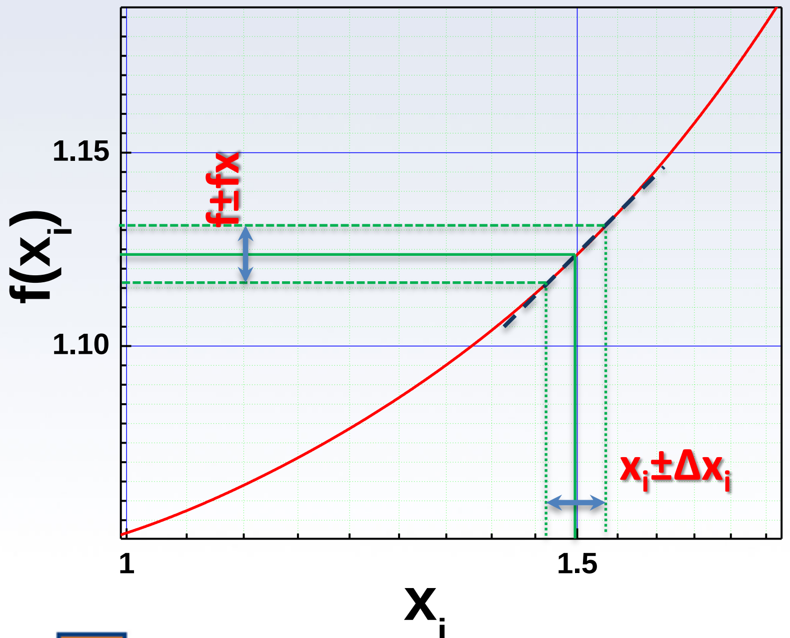


Data courtesy Tsung-Lin Hsieh, Physics 401, Fall 2011.

Appendix #4. Error propagation.

$$y = f(x1, x2 \ldots xn)$$

$$\Delta f(x_i, \Delta x_i) = \sqrt{\sum_{i=1}^{n} \left[\frac{\partial f}{\partial x_i}\right]^2 \cdot \Delta x_i^2}$$



Error propagation. Example.

Derive resonance frequency *f* from measured inductance *L±ΔL* and capacitance *C±ΔC*

$$f_0(L,C) = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$L_1 = 10 \pm 1\text{mH}, \quad C_1 = 10 \pm 2\mu\text{F}$$

$$\Delta f(L,C,\Delta L,\Delta C) = \sqrt{\left[\frac{\partial f}{\partial L}\right]^2 \cdot \Delta L^2 + \left[\frac{\partial f}{\partial C}\right]^2 \Delta C^2}$$

$$\frac{\partial f}{\partial L} = \frac{-1}{4\pi} C^{-\frac{1}{2}} L^{-\frac{3}{2}};$$

$$\frac{\partial f}{\partial C} = \frac{-1}{4\pi} L^{-\frac{1}{2}} C^{-\frac{3}{2}}$$

Results:

$f(L_1,C_1)$=503.29212104487Hz

Δf=56.26977Hz

$f(L_1,C_1)$=503±56Hz

Error propagation. Example.

$L_1 = 10 \pm 1\text{mH}, \quad C_1 = 10 \pm 1\mu\text{F}$ **Where these numbers are coming from?**

1. Using commercial resistors, capacitors, inductances…

C=500pF±5%

L=35mH±10%

2. Measuring the parameters using standard equipment

SENCORE "Z" meter model LC53
Capacitance measuring accuracy ±5%
Inductance measuring accuracy ±2%

Agilent E4980A Precision LCR Meter

Basic accuracy ±0.05%